Argumentation-Based Models of Agent Reasoning and Communication

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Outline

- Logic and Argumentation
  - Dung’s Theory of Argumentation
  - The Added Value of Argumentation
  - Rationality Postulates for Logic-based Argumentation

- Argumentation Based Dialogue
  - Argument Game Proof Theories
  - Generalisation to Dialogue
  - Schemes, Critical Questions and MAS applications
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- Logic and Argumentation
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  - Schemes, Critical Questions and MAS applications
Abstract Argumentation Theory and non-monotonic reasoning
A Dung argumentation framework \( \mathbf{AF} \) is a directed graph
\[
(\mathbf{Args}, \mathbf{Att})
\]
Where the nodes \( \mathbf{Args} \) denote arguments and \( \mathbf{Att} \) is a conflict-based binary attack relation between arguments.

Given a logic \( \mathcal{L} \) define:

1) What constitutes an argument
2) What constitutes an attack between two arguments
3) Given a set of wff \( \Delta \) in \( \mathcal{L} \) construct all the arguments and relate them by the attacks in an \( \mathbf{AF} \) (i.e., instantiate \( \mathbf{AF} \))

Logic Programming Instantiation of a Dung Argumentation Framework

\[ \Delta = \{ q : - p, \ \text{not } s ; s : - \text{not } g ; g : - m ; p ; m \} \]

- Given a set of wff in some logic \( \mathcal{L} \) define:

  1) What constitutes an argument

  \[ X = [q : - p, \ \text{not } s ; p] \quad \text{q is the claim of argument } X \]
  \[ Y = [s : - \text{not } g ] \quad \text{s is the claim of argument } Y \]
  \[ Z = [g : - m ; m] \quad \text{g is the claim of argument } Z \]
Logic Programming Instantiation of a Dung Argumentation Framework

\[ \Delta = \{ q : - p, \text{ not } s \ ; \\
   s : - \text{ not } g \ ; \\
   g : - m \\
   p ; \\
   m \} \]

- Given a set of wff in some logic \( \mathcal{L} \) define:

2) What constitutes an attack

\[ X = [q : - p, \text{ not } s ; p] \text{ and } Y = [s : - \text{ not } g ] \text{ and } Z = [g : - m ; m] \]

\((Y,X) \quad Att\)
Logic Programming Instantiation of a Dung Argumentation Framework

\[ \Delta = \{ q : \neg p, \neg s ; \\
    s : \neg g ; \\
    g : - m \\
    p ; \\
    m \} \]

- Given a set of wff in some logic \( \mathcal{L} \) define:
  1) What constitutes an attack

\[ X = [q : \neg p, \neg s ; p] \text{ and } Y = [s : \neg g] \text{ and } Z = [g : - m ; m] \]

\((Y,X) \quad \text{Att} \quad (Z,Y) \quad \text{Att}\)
Logic Programming Instantiation of a Dung Argumentation Framework

\[(\text{Args}, \text{Att}) = \]

\[Z \rightarrow Y \rightarrow X\]
Argument Evaluation

- AF = (Args, Att)

- What are the justified / rejected / undecided arguments?
Dung’s calculus of opposition

- Evaluation based on intuitive notion of reinstatement / defence

\[ S = X \]

\[ Y \]
Dung’s calculus of opposition

- Evaluation based on intuitive notion of reinstatement / defence

\[ S = X \rightarrow Z \]

- \( Z \) defends/reinstates \( X \) (\( X \) is acceptable w.r.t. \( S \))
Dung’s calculus of opposition

- Evaluation based on intuitive notion of reinstatement / defence

S = \[X \quad Z\]

- Z defends/reinstates X (X is acceptable w.r.t. S)

- If S is conflict free (contains no two arguments that attack), and all arguments in S are acceptable w.r.t. S, then S is admissible
Dung’s calculus of opposition

- Evaluation based on intuitive notion of reinstatement / defence

\[ S = X \quad [q : p, \text{not } s \mid p] \quad Z \quad [g : m \mid m] \]

- The set S of arguments is admissible since it is conflict free and all its contained arguments are defended against attacks
Dung semantics

Let $S$ be admissible:

- $S$ is a **complete** extension iff every argument acceptable w.r.t. $S$ is in $S$
- $S$ is the **grounded** extension iff it is the smallest **complete** extension
- $S$ is a **preferred** extension iff it is a maximal **complete** extension
- $S$ is a **stable** extension iff every argument not in $S$ is attacked by an argument in $S$

**Other semantics defined in the literature** *

- Semi-stable semantics
- Ideal Semantics
- etc

Example 1

Is $\emptyset$ admissible?
Is $\emptyset$ complete?

Is \{A\} admissible?
Is \{A\} complete?

Is \{A,D\} admissible?
Is \{A,D\} complete?

A $\rightarrow$ C $\rightarrow$ D

What are the grounded, preferred and stable extensions?
Example 2

Is $\emptyset$ admissible? Is \{A,C\} admissible?
Is $\emptyset$ complete?

Is \{A\} admissible?
Is \{A\} complete?

Is \{C\} admissible?
Is \{C\} complete?

A $\rightarrow$ C

What are the grounded, preferred and stable extensions?
Example 3

Is $\emptyset$ admissible?
Is \{A\} admissible?
Is \{A,B\} admissible?

What are the grounded, preferred and stable extensions?
Labelling Approach to Evaluating Extensions *

Given an \( AF = (Args, Att) \)
- \( X \in Args \) is IN iff \( (Y,X) \in Att \rightarrow Y \) is OUT
- \( X \in Args \) is OUT iff \( \exists (Y,X) \in Att \) such that \( Y \) is IN
- \( X \in Args \) is UNDEC iff \( \exists (Y,X) \in Att \) such that \( Y \) is UNDEC and \( \neg \exists (Y,X) \in Att \) such that \( Y \) is IN

Each framework can have many legal labellings
Legal labelling \textit{minimising} IN is grounded
Legal labelling \textit{maximising} IN is preferred
Legal labelling UNDEC = \( \emptyset \) is stable

Example 4

What are the preferred extensions? What is the grounded extension?

\{A, D\} and \{B, D\} are preferred extensions

\emptyset is grounded extension
Properties of Extensions

- Many properties of extensions have been studied, e.g.:
  - Each AF has a single grounded extension that is the intersection of all complete extensions
  - Each stable extension is preferred, but not vice versa
  - If $X$ is acceptable w.r.t. an admissible extension $E$, then $E \cup X$ is admissible (Fundamental Lemma)
The Justified Arguments of a Framework

X is *sceptically justified* under semantics E if X is in *all* E extensions

X is *credulously justified* under semantics E if X is in *at least one* E extension

\{A,D\} and \{B,D\} are preferred extensions

⇒ D is justified

∅ is grounded extension

⇒ no argument is justified
Argumentation-based Non-monotonic inference relation

- Abstract \( (Args, Att) \) defined by set of wff \( \Delta \) in logic \( \mathcal{L} \)

- \( \Delta \models_{AF} \alpha \) iff \( \alpha \) is the claim of a sceptically justified argument in \( Args \)

- Logic programming, default logic, auto-epistemic logic, defeasible logic, … all shown to conform to Dung’s semantics (alternative to model theoretic semantics ? Dialectical Semantics !)

  e.g.

  \[ \Delta \models_{LP} \alpha \text{ under well founded semantics iff } \Delta \models_{AF} \alpha \text{ under grounded semantics} \]
Argumentation-based characterisation of non-monotonic inference in logic programming

- Abstract \((\text{Args, Att})\) defined by set of wff \(\Delta\) in logic \(\mathcal{L}\)

- \(\Delta \models_{AF} \alpha\) iff \(\alpha\) is the claim of a sceptically justified argument in Args

- \(\Delta \models_{LP} \alpha\) under well founded semantics iff \(\Delta \models_{AF} \alpha\) under grounded semantics

\[
X = [q :- p, \text{not } s ; p] \quad Y = [s :- \text{not } g] \quad Z = [g :- m ; m]
\]

Grounded extension is \(\{X, Z\}\) and so \(\Delta \models_{AF} q, g\) corresponding to \(\Delta \models_{LP} q, g\)
Argumentation-based Non-monotonic inference relation

- Abstract \((\mathcal{A}, \mathcal{A})\) defined by set of wff \(\Delta\) in logic \(\mathcal{L}\)

- \(\Delta \vdash_{AF} \alpha\) iff \(\alpha\) is the claim of a sceptically justified argument in \(\mathcal{A}\)

- Define arguments and attacks from a possibly inconsistent set \(\Delta\) of wff in a **monotonic** logic \(\mathcal{L}\).
  
  Yields non-monotonic inference relation \(\Delta \vdash_{AF} \alpha\) thus resolving inconsistencies in underlying \(\Delta\)
Classical Logic-based Argumentation *

- Define arguments and attacks from a possibly inconsistent set \( \Delta \) of propositional classical wff

- A ClArg argument is a pair \((\Gamma, \alpha)\) such that

1) \( \Gamma \models_{\text{CL}} \alpha \)
2) \( \Gamma \) is consistent
3) No proper subset of \( \Gamma \) entails \( \alpha \)

- \((\Gamma, \alpha)\) attacks \((\Sigma, \beta)\) if \( \alpha \equiv \neg \delta \) for some \( \delta \in \Sigma \)

Classical Logic Argumentation: An Example

- Framework defined by $\Delta = (p, q, p \rightarrow \neg q)$

\[
\begin{align*}
\{ p \rightarrow \neg q \} : & p \rightarrow \neg q \\
\{ p, q \} : & p \land q \\
\{ q, p \rightarrow \neg q \} : & \neg p \\
\{ p, p \rightarrow \neg q \} : & \neg q \\
\{ p \} : & p \\
\{ q \} : & q
\end{align*}
\]
Classical Logic Argumentation: An Example

\{ p → ¬q \}: p → ¬q

\{ p , q \}: p ∧ q

\{ q , p → ¬q \}: ¬p ↔ \{ p , p → ¬q \}: ¬q

\{ p \}: p

\{ q \}: q

stable extension 1
Classical Logic Argumentation: An Example

\{ p \rightarrow \neg q \} : p \rightarrow \neg q

\{ p, q \} : p \land q

\{ q, p \rightarrow \neg q \} : \neg p \iff \{ p, p \rightarrow \neg q \} : \neg q

\{ p \} : p

\{ q \} : q

stable extension 2
Classical Logic Argumentation: An Example

\{ p \rightarrow \neg q \} : p \rightarrow \neg q

\{ p, q \} : p \land q

\{ q, p \rightarrow \neg q \} : \neg p \iff \{ p, p \rightarrow \neg q \} : \neg q

\{ p \} : p

\{ q \} : q

stable extension 3
Classical Logic Argumentation: An Example

\{ p \rightarrow \neg q \}: p \rightarrow \neg q

\{ p \land q \}: p \land q

\{ q \land p \rightarrow \neg q \}: \neg p \iff \{ p \land p \rightarrow \neg q \}: \neg q

\{ p \}: p \quad \{ q \}: q

- 3 preferred/stable extensions corresponding to three max consistent subsets of \{ p, q, p \rightarrow \neg q \}!

- No argument is in every extension (sceptically justified)
Argumentation-based Non-monotonic inference relation

- Abstract \((\text{Args}, \text{Att})\) defined by set of wff \(\Delta\) in logic \(\mathcal{L}\)

- \(\Delta \mid_{\text{AF}} \alpha\) iff \(\alpha\) is the claim of a sceptically justified argument inArgs
Classical Logic Argumentation : An Example

\{ p \rightarrow \neg q \} : p \rightarrow \neg q

\{ p, q \} : p \land q

\{ q, p \rightarrow \neg q \} : \neg p \iff \{ p, p \rightarrow \neg q \} : \neg q

\{ p \} : p

\{ q \} : q

- 3 preferred/stable extensions corresponding to three max consistent subsets of \{ p, q, p \rightarrow \neg q \} !

- So AF inference relation does not arbitrate between conflicting conclusions !

- When instantiating with monotonic logic what does argumentation do for you unless you have some way of arbitrating between conflicts ?
Preferences and Argumentation *

• One solution is to use preferences over arguments to arbitrate

• Partial ordering ≤ (preference relation) over arguments

\[(\text{Args,Att}, \leq)\]

• If X attacks Y and Y strictly preferred to X (X < Y) then X cannot be moved as a successful attack (defeat) on Y

• So based on ≤ and Att define a defeat relation Def over Args

• Extensions and justified arguments defined by (Args,Def)

Preferences and Argumentation

{ p \rightarrow \neg q } : p \rightarrow \neg q

{ p, q } : p \land q

\{ q, p \rightarrow \neg q \} : \neg p \iff \{ p, p \rightarrow \neg q \} : \neg q

\{ p \} : p
\{ q \} : q

\{ q, p \rightarrow \neg q \} : \neg p < \{ p \} : p
\{ p, p \rightarrow \neg q \} : \neg q < \{ q \} : q
Preferences and Argumentation

\[
\begin{align*}
\{ p \rightarrow \neg q \} : p \rightarrow \neg q \\
\{ p , q \} : p \land q \\
\{ q , p \rightarrow \neg q \} : \neg p \iff \{ p , p \rightarrow \neg q \} : \neg q \\
\{ p \} : p & \quad \{ q \} : q \\
\{ q , p \rightarrow \neg q \} : \neg p < \{ p \} : p \\
\{ p , p \rightarrow \neg q \} : \neg q < \{ q \} : q
\end{align*}
\]

Single stable extension of \((\text{Args},\text{Def}) = \{ \{ p \} : p , \{ q \} : q , \{ p , q \} : p \land q \}\)
Argumentation-based Non-monotonic inference relation

- $\Delta \models_{AF} \alpha$ iff $\alpha$ is the claim of a sceptically justified argument in Args

- $\Delta \models_{LP} \alpha$ under well founded semantics iff $\Delta \models_{AF} \alpha$ under grounded semantics

- Is there a non-monotonic inference relation that corresponds to the argumentation inference relation defined by classical logic argumentation with preferences?
Argumentation based characterisation of Brewka’s Non-monotonic Preferred Subtheories *

- Totally ordered set of propositional classical wff inducing a stratification:

\[
\begin{align*}
T_1 & \quad p, q \\
T_2 & \quad \neg p, s, \neg s \\
\vdots & \\
T_n &
\end{align*}
\]

- Start with maximal consistent subset of \(T_1\) then maximally consistently extend with formulae in \(T_2\) then ... all the way to \(T_n\)

- \(\Rightarrow\) many preferred subtheories – classical closure of formulae in intersection are non-monotonic inferences (\(\models_{ps}\))

Argumentation based characterisation of Brewka’s Non-monotonic Preferred Subtheories

- Totally ordered set of propositional classical wff inducing a stratification:

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\begin{align*}
T_1 & \quad \vdash p, q \\
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\vdots & \\
T_n & \\
\end{align*}
\]

- Start with maximal consistent subset of \( T_1 \) then maximally consistently extend with formulae in \( T_2 \) then ... all the way to \( T_n \)

- \( \rightarrow \) many preferred subtheories – classical closure of formulae in intersection are non-monotonic inferences (\( \models_{ps} \))

- E.g., \( \{p,q,s\} \) and \( \{p,q,\neg s\} \) \( \models_{ps} = Cn(p,q) \)
Argumentation based characterisation of Brewka’s Preferred Subtheories

- Build classical logic arguments from \{T_1, \ldots, T_n\}

- \(X < Y\) if there is premise in \(X\) that is strictly ordered below all premises in \(Y\) according to total ordering
  e.g. \(T_1 = \{p, q\}\), \(T_2 = \{p \rightarrow \neg q\}\)

\[
\{q, p \rightarrow \neg q\} : \neg p < \{p\} : p \quad \{p, p \rightarrow \neg q\} : \neg q < \{q\} : q
\]

- Evaluate justified arguments under stable semantics using argument preference ordering to determine defeats

Preferences and Argumentation

\[
\begin{align*}
\{ p \rightarrow \neg q \} & : p \rightarrow \neg q \\
\{ p, q \} & : p \land q \\
\{ q, p \rightarrow \neg q \} & : \neg p \iff \{ p, p \rightarrow \neg q \} : \neg q \\
\{ p \} & : p \\
\{ q \} & : q
\end{align*}
\]

\[
\{ q, p \rightarrow \neg q \} : \neg p < \{ p \} : p \\
\{ p, p \rightarrow \neg q \} : \neg q < \{ q \} : q
\]

Single stable extension of \((Args,Def) = \{ \{p\} : p, \{q\} : q, \{p,q\} : p \land q \}\)
Argumentation based characterisation of Brewka’s Preferred Subtheories

- Build classical logic arguments from \{T_1, \ldots, T_n\}

- \(X < Y\) if there is premise in \(X\) that is strictly ordered below all premises in \(Y\) according to total ordering
  
e.g. \(T_1 = \{p, q\}\), \(T_2 = \{p \rightarrow \neg q\}\)

- \(\{q, p \rightarrow \neg q\} : \neg p < \{p\} : p\) \(\{p, p \rightarrow \neg q\} : \neg q < \{q\} : q\)

- Evaluate justified arguments under stable semantics using argument preference ordering to determine defeats

- We *show* that \(\models_{ps} = \models_{AF}\)

Preferences and Argumentation

\[ \{ p \rightarrow \neg q \} : p \rightarrow \neg q \]

\[ \{ p, q \} : p \land q \]

\[ \{ q, p \rightarrow \neg q \} : \neg p \iff \{ p, p \rightarrow \neg q \} : \neg q \]

\[ \{ p \} : p \]

\[ \{ q \} : q \]

\[ \{ q, p \rightarrow \neg q \} : \neg p < \{ p \} : p \]

\[ \{ p, p \rightarrow \neg q \} : \neg q < \{ q \} : q \]

\[ \sim AF = \text{Cn}(p,q) \]

Single stable extension of \((\text{Args}, \text{Def}) = \{ \{p\} : p, \{q\} : q, \{p,q\} : p \land q \} \)
More on Abstract Argumentation

- So far we have seen how argumentation can define an inference relation over set of instantiating formulae.

- Other works in which argumentation used for decision making (e.g., L. Amgoud, H. Prade. *Using arguments for making and explaining decisions*. In: Artificial Intelligence (AIJ). V.173, pp. 413-436, 2009)
  - arguments for beliefs (epistemic) and decision options (practical) and evaluation makes use of decision principles.

- Extensions of abstract argumentation, e.g.,
  - values associated with arguments and ordering over values used to arbitrate amongst arguments (TJM Bench-Capon. *Persuasion in practical argument using value-based argumentation frameworks*. Journal of Logic and Computation 13 (3), 429-448)
  - AFs extended with arguments that attack attacks, so integrating argumentation-based reasoning about preferences (S. Modgil. Reasoning about preferences in Argumentation Frameworks. In: Artificial Intelligence (AIJ). V.173, 9-10, 2009.)
The Added Value of Argumentation
Abstract Argumentation

So what accounts for the popularity of argumentation. Who cares and why?

E.g. Logic Programming, Default Logic ...  
E.g. Preferred Subtheories
The Added Value (1)

- Basis for defining procedures for *distributed* non-monotonic reasoning based on simple, intuitive principle of reinstatement

\[ X = [q : p, not s ; p] \]

Ag1 logic program \(\Rightarrow\) \[ X = [q : p, not s ; p] \] Ag2 logic program
The Added Value (1)

- Basis for defining procedures for distributed non-monotonic reasoning based on simple, intuitive principle of reinstatement

\[ X = [q : \neg p, \neg s ; p] \]
\[ Y = [s : \neg g] \]

Ag1 logic program

Ag2 logic program
Basis for defining procedures for distributed non-monotonic reasoning based on simple, intuitive principle of reinstatement

\[ Z = [g : m ; m] \]
\[ X = [q : p, not s ; p] \]
\[ Y = [s : not g] \]
The Added Value (1)

- Basis for defining procedures for distributed non-monotonic reasoning based on simple, intuitive principle of reinstatement

- Argument Game proof theories $\Rightarrow$ basis for dialogues in which agents exchange arguments to persuade, deliberate over a course of action, negotiate ...

- Evaluation of exchanged arguments decides dialogue outcome

\[
X = \{ q : -p, \text{not } s ; p \} \\
Y = \{ s : -\text{not } g \} \\
Z = \{ g : -m ; m \}
\]
The Added Value (2)

- Reinstatement principle intuitive and familiar to human modes of reasoning and debate
- Argumentation based characterisations of computational reasoning understandable and accessible to human reasoning
- Abstractions that accommodate computational and human reasoning can provide bridging role so that:
  - Computational reasoning augments human reasoning
  - Human reasoning augments computational reasoning
  - Advancing AI through integrating human and computational reasoning


The Added Value (2)

- Reinstatement principle intuitive and familiar to human modes of reasoning and debate

- Argumentation based characterisations of computational reasoning understandable and accessible to human reasoning

- Abstractions that accommodate computational and human reasoning can provide bridging role so that:
  - Computational reasoning rationally informs human reasoning
  - Human reasoning augments computational reasoning
  - Advancing AI through integrating human and computational reasoning


Rationality Postulates
Rationality postulates *

- Given an \((\text{Args}, \text{Att})\) defined by set of wffs in logic \(L\) what properties would we rationally expect to hold of arguments contained in a complete extension \(E\) (remember grounded = smallest complete and preferred = maximal complete) ?

- **Consistency**: the claims of arguments in \(E\) are mutually consistent

- **Sub-argument Closure**: If \(X\) is an argument \(E\) then every sub-argument of \(X\) is in \(E\) (e.g., \([p]\) is a sub-argument of \([q :- p, \text{not } s ; p]\) )

- **Closure under Deductive (Strict) Inference**: If \(\beta_1\ldots \beta_n\) are claims of arguments in \(E\), and \(\beta_1\ldots \beta_n\) deductively entail \(\gamma\) then there is an argument in \(E\) with claim \(\gamma\)

---

Rationality postulates

- We want to specify under what conditions an \((\text{Args}, \text{Att})\) defined by set of wff \(\Delta\) in logic \(L\) satisfies the rationality postulates.

- But we have a dilemma. The abstract AF level is too abstract – we cannot refer to claims of arguments or sub-arguments ...

- On the other hand we don’t want to separately specify conditions for each individual logical instantiation (logic programming, classical logic etc) since one would like guidelines for logical instantiations of your choice!
Abstract Argumentation Framework

Instantiating Logic
The ASPIC+ framework *

Abstract level too abstract to study properties of argument extensions, e.g., are claims of arguments in an extension mutually consistent?

- ASPIC+ framework intermediate in abstraction – allows for broad range of instantiating logics and identifies conditions under which rationality postulates satisfied

Overview of ASPIC+

- Arbitrary language and generalised notion of conflict between wff (so that one can model classical negation $\neg$ or negation as failure $\textit{not}$). You are free to choose a language and declare when two formulae are in conflict.

- Arguments are trees built by chaining defeasible rules, strict inference rules and premises. But you are free to choose which defeasible(strict) rules and premises.

- Preferences over arguments. You are free to choose how preferences are defined.

- ASPIC+ identifies under what conditions the choices you make ensure rationality postulates satisfied.
b ⇒ f is a defeasible inference rule – ‘birds usually fly’
p → b is a strict inference rule – ‘penguins are without exception birds’
ASPIC+ Example Conditions for satisfaction of postulates

- If you have a strict inference $\beta_1, \beta_2 \ldots \beta_n \rightarrow \gamma$ then you must also have the strict inference rules:
  
  $\neg \gamma, \beta_2 \ldots \beta_n \rightarrow \neg \beta_1$
  
  $\beta_1, \neg \gamma, \ldots, \beta_n \rightarrow \neg \beta_2$
  
  $\vdots$
  
  $\beta_1, \beta_2 \ldots \neg \gamma \rightarrow \neg \beta_n$

- For example, if you have strict inference rule $p \rightarrow b$ then you must also have $\neg p \rightarrow \neg b$
ASPIC+ Example Conditions for satisfaction of postulates

- Attacks can only be directed at the conclusions of defeasible inference rules, and not at the conclusions of strict inference rules

\[
\begin{align*}
\{ p \rightarrow \neg q \} : p \rightarrow \neg q \\
\{ p, q \} : p \land q \\
\{ q, p \rightarrow \neg q \} : \neg p & \iff \{ p, p \rightarrow \neg q \} : \neg q \\
\{ p \} : p & \quad \{ q \} : q
\end{align*}
\]
ASPIC+ Example Conditions for satisfaction of postulates

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\{ p, q \} : & \ p \land q \\
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\{ q, p \rightarrow \neg q \} & : \neg p \\
\{ p, p \rightarrow \neg q \} & : \neg q \\
\{ p \} & : p \\
\{ q \} & : q
\end{align*}
\]
Suppose we allowed attacks on the conclusions of strict inference rules.

\[
\begin{align*}
\{ p \rightarrow \neg q \} : & \quad p \rightarrow \neg q \\
\{ p, q \} : & \quad p \land q \\
\{ q, p \rightarrow \neg q \} : & \quad \neg p \quad \iff \quad \{ p, p \rightarrow \neg q \} : \quad \neg q \\
\{ p \} : & \quad p \\
\{ q \} : & \quad q
\end{align*}
\]
ASPIC+ Example Conditions for satisfaction of postulates

- Suppose we allowed attacks on the conclusions of strict inference rules

\[
\{ p \rightarrow \neg q \} : p \rightarrow \neg q
\]

\[
\{ p, q \} : p \land q
\]

\[
\{ q, p \rightarrow \neg q \} : \neg p \quad \leftrightarrow \quad \{ p, p \rightarrow \neg q \} : \neg q
\]

\[
\{ p \} : p \\
\{ q \} : q
\]
Can anyone see why consistency would be violated?

\[
\begin{align*}
\{ p \rightarrow \neg q \} & : p \rightarrow \neg q \\
\{ p, q \} & : p \land q \\
\{ q, p \rightarrow \neg q \} & : \neg p \leftrightarrow \{ p, p \rightarrow \neg q \} : \neg q \\
\{ p \} & : p \\
\{ q \} & : q
\end{align*}
\]
ASPIC+ Example Conditions for satisfaction of postulates

- There is a complete and stable extension containing arguments with mutually inconsistent conclusions

\[
\begin{align*}
\{ p \rightarrow \neg q \} : p \rightarrow \neg q \\
\{ p, q \} : p \land q \\
\{ q, p \rightarrow \neg q \} : \neg p & \iff \{ p, p \rightarrow \neg q \} : \neg q \\
\{ p \} : p & \iff \{ q \} : q
\end{align*}
\]
ASPIC+ Example Conditions for satisfaction of postulates

\[
\begin{align*}
\{ p \rightarrow \neg q \} &: p \rightarrow \neg q \\
\{ p, q \} &: p \land q \\
\{ q, p \rightarrow \neg q \} &: \neg p &\iff \{ p, p \rightarrow \neg q \} &: \neg q \\
\{ p \} &: p \\
\{ q \} &: q
\end{align*}
\]

Things can go wrong with preferences. Suppose:

\[
\begin{align*}
\{ q, p \rightarrow \neg q \} &: \neg p &<& \{ p \} &: p \\
\{ p, p \rightarrow \neg q \} &: \neg q &<& \{ q \} &: q \\
\{ p, q \} &: p \land q &<& \{ p \rightarrow \neg q \} &: p \rightarrow \neg q
\end{align*}
\]
ASPIC+ Example Conditions for satisfaction of postulates

\{ p \rightarrow \neg q \} : p \rightarrow \neg q

\{ p, q \} : p \land q

\{ q, p \rightarrow \neg q \} : \neg p \iff \{ p, p \rightarrow \neg q \} : \neg q

\{ p \} : p

\{ q \} : q

Attacks do not succeed as defeats

\{ q, p \rightarrow \neg q \} : \neg p < \{ p \} : p

\{ p, p \rightarrow \neg q \} : \neg q < \{ q \} : q

\{ p, q \} : p \land q < \{ p \rightarrow \neg q \} : p \rightarrow \neg q
ASPIC+ Example Conditions for satisfaction of postulates

{ p → ¬q } : p → ¬q

{ p , q } : p ∧ q

{ q , p → ¬q } : ¬p ↔ { p , p → ¬q } : ¬q

{ p } : p

{ q } : q

Stable extension of (Args,Def) now contains arguments with inconsistent conclusions

{ q , p → ¬q } : ¬p < { p } : p

{ p , p → ¬q } : ¬q < { q } : q

{ p , q } : p ∧ q < { p → ¬q} : p → ¬q
ASPIC+ Example Conditions for satisfaction of postulates

{ \{ p \rightarrow \neg q \} : p \rightarrow \neg q \\
{ \{ p , q \} : p \land q \\
{\{ q , p \rightarrow \neg q \} : \neg p \iff \{ p , p \rightarrow \neg q \} : \neg q \\
{ \{ p \} : p \quad { \{ q \} : q \\

But if preference relation satisfies property of being *reasonable* then all three strict preferences are not possible (for otherwise it would result in a cycle in < )

{ \{ q , p \rightarrow \neg q \} : \neg p < { \{ p \} : p \\
{ \{ p , p \rightarrow \neg q \} : \neg q < { \{ q \} : q \\
{ \{ p , q \} : p \land q < { \{ p \rightarrow \neg q \} : p \rightarrow \neg q
ASPIC+ Example Conditions for satisfaction of postulates

\( \{ p \rightarrow \neg q \} : p \rightarrow \neg q \)

\( \{ p, q \} : p \land q \)

\( \{ q, p \rightarrow \neg q \} : \neg p \iff \{ p, p \rightarrow \neg q \} : \neg q \)

\( \{ p \} : p \quad \{ q \} : q \)

If

\( \{ q, p \rightarrow \neg q \} : \neg p \lt \{ p \} : p \)

\( \{ p, p \rightarrow \neg q \} : \neg q \lt \{ q \} : q \)

then

\( \{ p, q \} : p \land q \not\lt \{ p \rightarrow \neg q \} : p \rightarrow \neg q \)
ASPIC+ Example Conditions for satisfaction of postulates

\[
\begin{align*}
\{ p \rightarrow \neg q \} : p \rightarrow \neg q \\
\{ p, q \} : p \land q \\
\{ q, p \rightarrow \neg q \} : \neg p & \iff \{ p, p \rightarrow \neg q \} : \neg q \\
\{ p \} : p & \quad \{ q \} : q
\end{align*}
\]

If
\[
\begin{align*}
\{ q, p \rightarrow \neg q \} : \neg p & < \{ p \} : p \\
\{ p, p \rightarrow \neg q \} : \neg q & < \{ q \} : q
\end{align*}
\]

then
\[
\{ p, q \} : p \land q \not\leq \{ p \rightarrow \neg q \} : p \rightarrow \neg q
\]
ASPIC+ Example Conditions for satisfaction of postulates

\[ \{ p \rightarrow \neg q \} : p \rightarrow \neg q \]
\[ \{ p, q \} : p \land q \]
\[ \{ q, p \rightarrow \neg q \} : \neg p \iff \{ p, p \rightarrow \neg q \} : \neg q \]
\[ \{ p \} : p \]
\[ \{ q \} : q \]
\[ \{ q, p \rightarrow \neg q \} : \neg p < \{ p \} : p \]
\[ \{ p, p \rightarrow \neg q \} : \neg q < \{ q \} : q \]

Single stable extension of \((\text{Args}, \text{Def})\) = \{ \{ p \} : p, \{ q \} : q, \{ p, q \} : p \land q \}
I’ve glossed over many details, but take home message is:

ASPIC+ is a general framework that allows for a broad range of possible logical instantiations, and provides guidelines for your choice of inference rules, how you define attacks, how you define preferences etc, so that you can be sure that your logical instantiation of Dung frameworks with preferences satisfies rationality postulates.

More papers


Summary

- Dung’s abstract theory of argumentation and example logical instantiations (with preferences)

- Correspondences between non-monotonic inference relation of instantiating logic and inference relation defined by claims of justified arguments

- The added value of argumentation – generalisation to dialogue (distributed reasoning) and familiar principles in everyday reasoning and debate

- Rationality postulates and ASPIC+